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Heavy Quarks in DIS (Theory)

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Abstract. I review the various methods for taking account of finite quark masses in DIS and related processes. I pay particular attention to the so-called variable flavour number schemes (VFNS) which are designed to extrapolate smoothly from the region near threshold of production of heavy quark pairs to the region where the quarks become effectively massless.

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1. Introduction

In the past few years there has been a considerable effort to improve the theoretical description of the effects due to heavy quarks (i.e $m_H \gg \Lambda_{QCD}$) in DIS and related scattering processes. As is often the case this has been driven by the vast improvement in experimental data rendering the previous simplistic theoretical description inadequate. There is now not only data on the charm component of the structure function at relatively high x ($x > 0.01$) from the EMC collaboration [1], but also at very low x ($0.00005 < x < 0.01$) from HERA [2,3]. This latter data also exists not only in terms of the total inclusive cross-section, but also in more differential forms, which is presented by J Cole [4] in the experimental plenary talk. Moreover, the charm component to the total structure function now comprises about 25% of the total at the highest y values, and with the extremely precise data now available a more sophisticated treatment of heavy quark effects is needed here. Perhaps surprisingly, this is also the case when comparing the theoretical predictions to the NMC data [5] at $x \sim 0.05$ and $Q^2 \sim 10\text{GeV}^2$, since although charm only constitutes 5 – 10% of the total structure function it contributes more strongly to the value of $dF_2/d\ln Q^2$, which is the quantity we are really testing in fits to data.

2. Theoretical Methods

The oldest, and by far the simplest method for treating heavy flavors in DIS (and other processes) is known as the zero mass variable flavour scheme (ZM-VFNS). Using this method one treats the heavy quark as being infinitely massive below some scale μ_T , i.e. it decouples from the theory below this scale, and as completely massless above μ_T . Therefore the number of quarks flavors just changes from n_f to n_{f+1} in the massless expressions for splitting functions, coefficient functions etc. at some scale usually chosen as $\mu_T^2 = m_H^2$. It seems intuitively obvious that if the evolution of the massive quark starts at $\mu^2 = m_H^2$, it should evolve from a value of roughly zero at this scale. However, this is hardly quantitative, and the real content of the ZM-VFNS is the set of matching conditions required to guarantee that the correct results are obtained for both $Q^2/m_H^2 \rightarrow 0$ and $Q^2/m_H^2 \rightarrow \infty$ [6]. This results in a prescription for how the coupling should change at the matching point, and also for the form of the parton distributions at this point. In fact one can calculate the partons in the massless $n_f + 1$ flavour scheme entirely in terms of those in the n_f flavour scheme, i.e.

$$f_k^{n_f+1}(\mu^2, m_H^2) = A_{kj}(\mu^2/m_H^2) \otimes f_j^{n_f}(\mu^2), \quad (1)$$

where the matrix elements are perturbatively calculable, and can be found to $\mathcal{O}(\alpha_s^2)$ in [7]. This expression is correct up to higher twist ($\mathcal{O}(\Lambda_{QCD}^2/m_H^2)$) corrections due to the intrinsic heavy quark distribution. It turns out that if we do choose precisely $\mu_T^2 = m_H^2$ then to NLO the heavy quark distribution does start from zero. At NNLO, i.e. $\mathcal{O}(\alpha_s^2)$, this is no longer true, but the non-zero starting distribution is known [7]. The ZM-VFNS has been used in global fits to structure function data for many years. However, it has a clear deficiency, i.e. the threshold treatment. It takes no account of the fact that quark pairs can only be created for $W^2 = Q^2(1/x - 1) \geq 4m_H^2$, and indeed has an error of $\mathcal{O}(m_H^2/Q^2)$ reflecting this. So until we reach $Q^2 \gg m_H^2$, it is not quantitatively useful, and is clearly not sufficient to describe much of the data now available.

In some senses the opposite to the ZM-VFNS is the fixed flavour number scheme (FFNS). In this scheme only those quarks with $m_H \leq \Lambda_{QCD}$ are treated as partons. All heavy quark production is generated via the perturbatively calculable coefficient functions describing the scattering of initial light partons. The finite quark mass effects are kept in the calculation of these coefficient functions, and hence the threshold behaviour for quark pair production is described correctly. The error on the structure function calculations in this scheme is $\mathcal{O}(\Lambda_{QCD}^2/m_H^2)$, reflecting the decoupling of the intrinsic heavy quark effects from the light parton distributions. This prescription thus works very well near the threshold region. Its only real potential problem is in the region of high Q^2 . This is because the perturbative series for the coefficient functions behaves like (letting $\mu^2 = Q^2$ from now on)

$$C(\alpha_s(Q^2), m_H^2/Q^2) = \sum_{n=1}^{\infty} \sum_{p=q}^n \alpha_s^n(Q^2) \ln^{n-p}(Q^2/m_H^2), \quad (2)$$

when $Q^2/m_H^2 \rightarrow \infty$, and q is a process dependent number ($q = 0$ for $F_2(x, Q^2)$ but $q = 1$ for $F_L(x, Q^2)$). Thus, a fixed order in α_s calculation is not guaranteed to give a good representation of the correct result at high Q^2 , and it would be desirable to

sum the series in leading powers of $\ln(Q^2/m_H^2)$. This is precisely what is done in actually treating the heavy quark as a parton in the ZM-VFNS, and determining its evolution by solving the renormalization group equations. Hence, although the FFNS seems preferable to the ZM-VFNS, it would seem, at least in principle, desirable to devise a scheme which contains the best parts of both the ZM-VFNS and the FFNS. Indeed, this is necessary if one actually wishes to consider parton distributions for heavy flavors without resigning oneself to large errors near threshold.

Various attempts have been made to achieve this objective, and the procedures are generally known as variable flavour number schemes. This is because they all have in common the features that for low scales, usually $Q^2 \leq m_H^2$, they are identical to the FFNS, having 3 partonic quarks, while for $Q^2/m_H^2 \rightarrow \infty$ the heavy quark is treated as a massless parton, evolving according to the massless evolution equations. There are at least 4 different versions of a VFNS currently available. One, of these, devised by Buza *et al.* [8], is at the level of structure functions rather than parton distributions, extrapolating smoothly from $F^{FFNS}(x, Q^2)$ at low Q^2 to $F^{ZM-VFNS}(x, Q^2)$ at high Q^2 via the subtraction of $F^{ASYM}(x, Q^2)$ (the FFNS result with all $\mathcal{O}(m_H^2/Q^2)$ corrections removed), which is the limit of the other two forms as $Q^2/m_H^2 \rightarrow 1$ or $\rightarrow \infty$. Another, the MRRS scheme [9], involves mass corrections to the evolution of the partons. Despite the fact that these disappear as $Q^2/m_H^2 \rightarrow \infty$, their presence in the evolution means that neither the parton distributions or the coefficient functions are precisely as they would be in the ZM-VFNS, even at asymptotic Q^2 . In this review I will concentrate on those schemes where the treatment of the parton distributions is exactly as in the ZM-VFNS, and hence where the theories tend unambiguously to this asymptotic limit.

The first of these schemes is known as the ACOT scheme [10]. It is defined by calculating the coefficient functions with both light and heavy initial quarks. The divergences in logs of (Q^2/m_H^2) are then factored in the same manner as the infrared divergences due to light initial states, and the heavy parton distributions defined via this procedure are guaranteed to obey the massless evolution equations, i.e. they are precisely the parton distributions defined in eq. (1). The coefficient functions remaining are guaranteed to be finite, and in fact identical to the massless coefficient functions in the limit $Q^2/m_H^2 \rightarrow \infty$. However, they contain all the mass effects important at lower Q^2 . At all orders this method will give exactly the same results as the FFNS, but the way of ordering the expansion is now somewhat different. The coefficient functions are related to those in the FFNS by this exact equality of the structure functions calculated via the two methods:

$$\begin{aligned} F(x, Q^2) &= C_k^{FFNS}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2) \\ &= C_j^{VFNS}(Q^2/m_H^2) \otimes A_{jk}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2), \end{aligned} \quad (3)$$

i.e. the $A_{jk}(Q^2/m_H^2)$ are the parts subtracted out of the divergent coefficient functions which depend on the massive quarks. That this procedure is well-defined to all orders was recently formally proved by Collins [11]. This scheme has a well-defined method of calculating the mass-dependent coefficient functions in terms of diagrams with initial massive quarks (for computational simplicity it is suggested in [11] that coefficient functions with initial heavy quarks be calculated in the massless limit) and the unique matrix elements $A_{jk}(Q^2/m_H^2)$. The only potential weakness of the approach is that coefficient functions now exist describing the scattering of a single heavy quark. Such coefficient functions, e.g. the zeroth order $c + \gamma^* \rightarrow c$, allow the

creation of final state heavy quarks for $W^2 < 4m_H^2$, and hence the correct kinematical threshold is not respected order by order in the coefficient functions. The factorization procedure guarantees that the correct threshold behaviour is regained, but this is due to cancellations between orders, and near threshold these cancellations can be larger than the final result, as seen in [10]. Hence the ordering of terms in the ACOT scheme must be different to that for the calculation of light quark structure functions in order to obtain the most accurate results.

More recently a different scheme has been proposed [12]. This takes the formal result of eq.(3), and notes that since there is one more VFNS coefficient function than there are FFNS coefficient functions, the former are not defined unambiguously. Therefore, it is decided to define the VFNS coefficient functions not from the diagrammatic representation, but by solving eq.(3) subject to the physical constraint that the derivative of the structure function is continuous (in the very dominant gluon sector) across the matching point at $Q^2 = m_H^2$. This results in heavy flavour coefficient functions which are related to the $\ln Q^2$ derivatives of $C_g^{FFNS,n}(x, Q^2/m_H^2)$ via the formal inverse of the LO quark-gluon splitting function, e.g. at leading order

$$C_{2,HH}^{VFNS,0}(Q^2/m_H^2) \otimes p_{qg}^0 = \frac{\partial C_{2,g}^{FFNS,1}(x, Q^2/m_H^2)}{\partial \ln Q^2}. \quad (4)$$

Hence, at the loss of a clear diagrammatic interpretation for the coefficient functions one gains the correct type of threshold dependence in all coefficient functions. In this scheme the ordering of the calculation is the same as for light partons. To all orders the two VFNSs are identical, and have identical prescriptions for the partons at each order. They are effectively related by a scheme change for coefficient functions which does not alter the parton distributions. There are in principle an infinite number of choices for such schemes.

However, the ambiguity in the VFNS coefficient functions only strictly occurs because we are working in the limit where the $n_f + 1$ flavour partons are completely determined by the n_f flavour distributions. (This is assumed in most practical uses of either of the VFNSs in global fits.) This assumption leads to a minimum error of $\mathcal{O}(\Lambda_{QCD}^2/m_H^2)$ due to the neglect of the effects of the intrinsic quark distribution (which though formally small, may be enhanced at large x [13]) and its influence on the other parton distributions. Collins has shown that within the ACOT scheme if one takes account of such effects, i.e. allows deviation from eq.(1) of this order, then the calculation of the structure functions becomes accurate to $\mathcal{O}(\Lambda_{QCD}^2/Q^2)$ [11]. Allowing this intrinsic heavy quark distribution formally removes the redundancy in the definition of the coefficient functions, and the FFNS and the alternative VFNS will now longer be identical to the ACOT scheme if summed to all orders. In the FFNS a similar accuracy can only be obtained by adding intrinsic heavy quark effects in some *ad hoc* manner. However, the accuracy in the alternative VFNS is in fact automatically of the same order as the ACOT scheme. To appreciate this, let the parton distributions, including the intrinsic heavy quarks, be identical in the two schemes. The coefficient functions at each order are different by $\mathcal{O}(m_H^2/Q^2)$. Nevertheless, this difference has been constructed to be unimportant when combining with the part of the parton distributions insensitive to the intrinsic heavy quarks. The remainder of the parton distributions is of $\mathcal{O}(\Lambda_{QCD}^2/m_H^2)$, and when combined with the difference in coefficient functions leads to a difference in structure functions of $\mathcal{O}(\Lambda_{QCD}^2/Q^2)$, i.e. the same order as the accuracy of the ACOT scheme. Again, this will be true for the whole

family of schemes discussed above. Hence all VFNSs have in principle the same type of accuracy, and none is guaranteed to be theoretically preferable.

3. Results

A comparison of the neutral current charm structure function obtained using the ZM-VFNS, the FFNS and the VFNS using the same input partons, and all at NLO is shown in fig.1. The biggest difference is between the ZM-VFNS and the others at small Q^2 , as we might expect. The differences between the VFNS and FFNS can be up to $\sim 20\%$. The failure of the ZM-VFNS at low Q^2 should surely mean that it is no longer used, except when its defects are not important. The FFNS is still used in many circumstances, including the GRV parameterizations [14], and it is clear that for the neutral current structure functions its use does not lead to any obvious problems. Indeed, GRV show that the variation due to factorization/renormalization scale changes is similar to that seen between the different schemes in fig.1. However, despite this evidence the FFNS does have potential shortcomings at high Q^2 . Indeed, it has been demonstrated in [15] that the charged current structure functions $F_{3,c}^{e^{+(-)}p}(x, Q^2)$ require the summation of large logs in (Q^2/m_H^2) in order to obtain a stable result. This may well be true for other processes, and the universality of parton distributions may be compromised in this scheme.

For the moment most of the relevant experimental data is for the neutral current structure function $F_2(x, Q^2)$. The direct data on the charm structure function is not yet accurate enough to discriminate between the approaches, and all work quite well (except for the ZM-VFNS for $Q^2 \sim m_c^2$). However, the large amount of accurate data on the total structure function does seem to be a discriminant. In both [12] and a CTEQ fit [16] it was found that the respective VFNS fits were superior to either the ZM-VFNS or the FFNS, with the latter faring worst. The improvement is seen not only for the HERA data but also for the NMC data. It is difficult to make a qualitative comparison with the GRV FFNS calculation since they do not really perform a best fit, and hence produce no χ^2 values. Each of the above procedures is also easily applicable to the case of charged current scattering as well as neutral current, and as a further test it would be interesting to see whether they result in any significant difference for the strange component of the sea quarks extracted by the CCFR collaboration [17].

Hence, it is clear that there are a variety of different methods for dealing with heavy flavors in DIS. Each have their own virtues and there presently seems to be no overwhelming reason for choosing one exclusively, though I would argue that a VFNS is the most sound theoretically. In the working group session it was also demonstrated that each scheme could be subject to relatively important further corrections in the form of summation of large threshold logarithms [18]. Further quantitative analysis of such effects using the techniques developed would lead to an even better understanding of heavy flavour production in DIS and supplement the significant progress made in this field in recent years. Also, the theoretical treatment is still a little behind the experimental development, and it would be useful to generalize the VFNS approaches to the situations where more differential inclusive cross-sections, such as the p_T - or rapidity distributions of the final state heavy quarks are described, and where at present there are only massless and FFNS [19] results. Hence, I conclude that in the theoretical treatment of heavy flavour physics for DIS significant progress has recently

been made, but there are many topics still to be investigated before we can claim to have a really detailed understanding.

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Figure captions

Figure 1. Charm quark structure function, $F_{2,c}(x, Q^2)$ for $x = 0.05$ and $x = 0.005$ calculated using the NLO VFNS (solid line) FFNS (dotted line) and ZM-VFNS (dashed line) with the same parton distributions and scale choice ($\mu^2 = Q^2$) in each case.

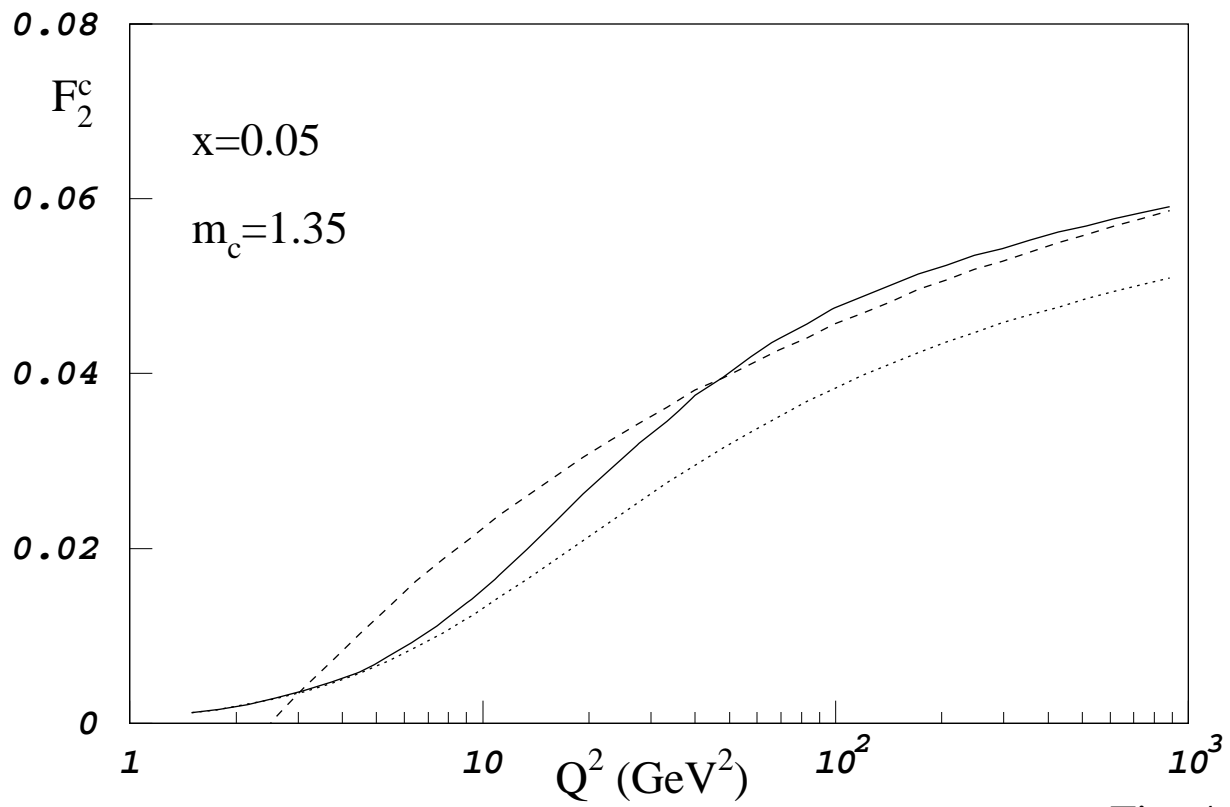
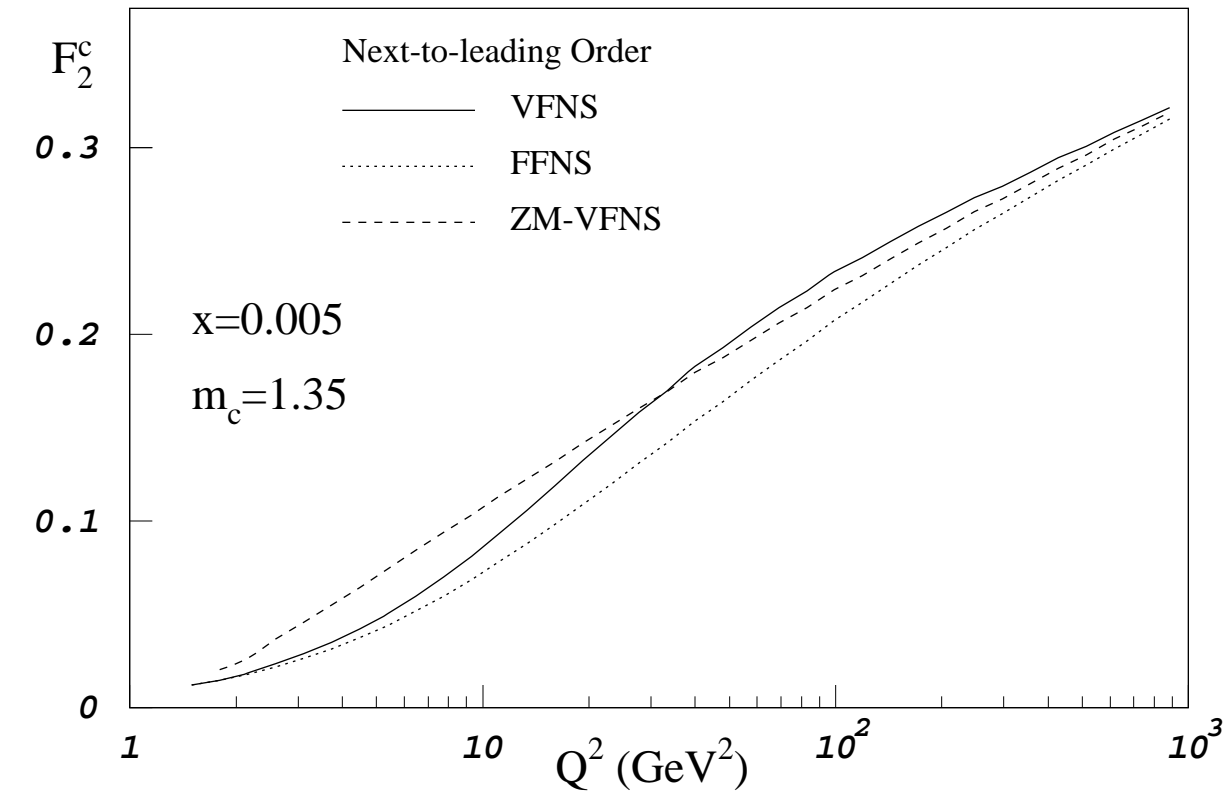


Fig. 4